Learning Intentions

- To learn how to calculate the expected value of an experiment.

Notes

1. The expected value is:

the long-term average outcome from a given experiment

2. The expected value is calculated by:

multiplying the value of each outcome by its probability.

3. Mr. Renwick rolls a fair 6-sided die. If he rolls a 4 or less, he receives no prize. If he rolls a 5 or more, he receives $10. What is the expected value from rolling the die once?

1 - $0
2 - $0
3 - $0
4 - $0
5 - $10
6 - $10

$$E = \frac{1}{6} \times 0 + \frac{1}{6} \times 0 + \frac{1}{6} \times 0 + \frac{1}{6} \times 0 + \frac{1}{6} \times \$10$$
$$+ \frac{1}{6} \times \$10$$

$$E = \frac{\$20}{6} = \$3.33$$

4. A lottery has 2,000 tickets. There is a $1,000,000 grand prize, and 5 $5,000 prizes. What is the expected value from buying a ticket?

$$E = \frac{1}{2000} \times \$1,000,000 + \frac{5}{2000} \times \$5,000 = \$512.50$$

5. There is a weighted die that has a 1/10 chance of landing on 1 or 2, a 1/5 chance of landing on 3 or 4, a 2/5 chance of landing on 5, and a 0% chance of landing on 6. A carnival game is played where you receive $10 times the number that appears on the die.

a. What is the expected value of the game?

1 - $10
2 - $20
3 - $30
4 - $40
5 - $50
6 - $60
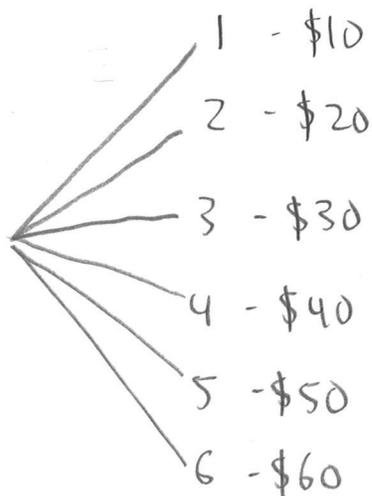
$$E = \frac{1}{10} \times \$10 + \frac{1}{10} \times \$20 + \frac{1}{5} \times \$30 + \frac{1}{5} \times \$40$$
$$+ \frac{2}{5} \times \$50 + 0 \times \$60$$

$$E = \$37$$

b. How much should the carnival charge to play the game?

At least $37

Questions

For all of the following questions, assume that it is the first round of Yahtzee and so all of the scoring categories are available.

1. After the second roll, you have 6 6 6 6 1. You decide to roll the 1 for your last roll. What is the expected value of your hand?

2. After the second roll, you have 1 1 2 3 4. You decide to roll one of the 1's for your last roll. What is the expected value of your hand?

3. After the second roll, you have 3 3 4 5 6. You decide to roll one of the 3's for your last roll. What is the expected value of your hand?

4. After the second roll, you have 2 3 4 5 5. You decide to roll one of the 5's for your last roll. What is the expected value of your hand?

5. Of the previous 3 hands, which has the highest expected value? Why?

6. After the second roll, you have 1 2 4 4 4. You decide to roll the 1 for your last roll, going for a full house. What is the expected value of your hand?

7. After the second roll, you have 1 2 4 4 4. You decide to roll the 1 and 2 for your last roll, going for a full house. What is the expected value of your hand?

8. Of the previous 2 hands, which has the highest expected value? Why?

1. 6 6 6 6 1

1 — 6 6 6 6 1 — 4 of a kind/chance - 25 points
2 — 6 6 6 6 2 — " " " - 26
3 — 6 6 6 6 3 — " " " 27
4 — 6 6 6 6 4 — " " " 28
5 — 6 6 6 6 5 — " " " 29
6 — 6 6 6 6 6 — Yahtzee - 50

$$E = \frac{1}{6} \times 25 + \frac{1}{6} \times 26 + \frac{1}{6} \times 27 + \frac{1}{6} \times 28 + \frac{1}{6} \times 29 + \frac{1}{6} \times 50$$

$$E = \frac{185}{6} = 30.83 \text{ points}$$

2. 1 1 2 3 4

1 - 1 1 2 3 4 - small straight - 30

2 - 1 2 2 3 4 - " " 30

3 - 1 2 3 3 4 - " " 30

4 - 1 2 3 4 4 - " " 30

5 - 1 2 3 4 5 - large straight 40

6 - 1 2 3 4 6 - small straight 30

$$E = \frac{1}{6} \times 30 + \frac{1}{6} \times 30 + \frac{1}{6} \times 30 + \frac{1}{6} \times 30 + \frac{1}{6} \times 40 + \frac{1}{6} \times 30$$

$$E = \frac{95}{3} = 31.67 \text{ points}$$

3. 3 3 4 5 6

1 - 1 3 4 5 6 - small straight - 30
2 - 2 3 4 5 6 - large straight - 40
3 - 3 3 4 5 6 - small straight - 30
4 - 3 4 4 5 6 -   "   "   - 30
5 - 3 4 5 5 6 -   "   "   - 30
6 - 3 4 5 6 6 -   "   "   - 30

$E = \frac{1}{6} \times 30 + \frac{1}{6} \times 40 + \frac{1}{6} \times 30 + \frac{1}{6} \times 30 + \frac{1}{6} \times 30 + \frac{1}{6} \times 30$

$E = \frac{95}{3} = 31.67$ points

4. 2 3 4 5 5

1 - 1,2 3 4 5 - large straight - 40
2 - 2 2 3 4 5 - small " - 30
3 - 2 3 3 4 5 - " " - 30
4 - 2 3 4 4 5 - " " - 30
5 - 2 3 4 5 5 - " " - 30
6 - 2 3 4 5 6 - large " - 40

$$E = \frac{1}{6} \times 40 + \frac{1}{6} \times 30 + \frac{1}{6} \times 30 + \frac{1}{6} \times 30 + \frac{1}{6} \times 30 + \frac{1}{6} \times 40$$

$$E = \frac{100}{3} = 33.3 \text{ points}$$

5. $E(1 1 2 3 4) = 31.67$ ← lower - only a 5 gives a large straight

$E(3 3 4 5 6) = 31.67$ ← lower - " " 2 " " " "

$E(2 3 4 5 5) = 33.3$ ← highest, because a 1 or a 6 gives a large straight

6. 1 2 4 4 4

1 - 1 2 4 4 4 - 3 of a kind/chance - 15
2 - 2 2 4 4 4 - full house - 25
3 - 2 3 4 4 4 - 3 of a kind/chance - 17
4 - 2 4 4 4 4 - 4 of a kind/chance - 18
5 - 2 4 4 4 5 - 3 of a kind/chance - 19
6 - 2 4 4 4 6 - " " " " " - 20

$E = \frac{1}{6} \times 15 + \frac{1}{6} \times 25 + \frac{1}{6} \times 17 + \frac{1}{6} \times 18 + \frac{1}{6} \times 19 + \frac{1}{6} \times 20$

$E = 19$ points

7. 1 2 4 4 4

	1	2	3	4	5	6
1	1 1 4 4 4 Full house 25	1 2 4 4 4 3 of a kind 15	1 3 4 4 4 3 of a kind 16	1 4 4 4 4 4 of a kind 17	1 4 4 4 5 3 of a kind 18	1 4 4 4 6 3 of a kind 19
2	1 2 4 4 4 3 of a kind 15	2 2 4 4 4 Full house 25	2 3 4 4 4 3 of a kind 17	2 4 4 4 4 4 of a kind 18	2 4 4 4 5 3 of a kind 19	2 4 4 4 6 3 of a kind 20
3	1 3 4 4 4 3 of a kind 16	2 3 4 4 4 3 of a kind 17	3 3 4 4 4 Full house 25	3 4 4 4 4 4 of a kind 19	3 4 4 4 5 3 of a kind 20	3 4 4 4 6 3 of a kind 21
4	1 4 4 4 4 4 of a kind 17	2 4 4 4 4 4 of a kind 18	3 4 4 4 4 4 of a kind 19	4 4 4 4 4 Yahtzee 50	4 4 4 4 5 4 of a kind 21	4 4 4 4 6 4 of a kind 22
5	1 4 4 4 5 3 of a kind 18	2 4 4 4 5 3 of a kind 19	3 4 4 4 5 3 of a kind 20	4 4 4 4 5 4 of a kind 21	4 4 4 5 5 Full house 25	4 4 4 5 6 3 of a kind 23
6	1 4 4 4 6 3 of a kind 19	2 4 4 4 6 3 of a kind 20	3 4 4 4 6 3 of a kind 21	4 4 4 4 6 4 of a kind 22	4 4 4 5 6 3 of a kind 23	4 4 4 6 6 Full house 25

$$E = \frac{1}{36}[25+15+16+17+18+19+15+25+17+18+19+20+16+17+25+19+20+21+$$
$$17+18+19+50+21+22+18+19+20+21+25+23+19+20+21+22+23+25]$$

$$E = \frac{745}{36} = 20.7 \text{ points}$$