Name: ______________

Learning Intentions

- To learn how the physics of uniform circular motion and gravitational attraction combine to describe the orbit of planets and satellites

Notes

1. Astronomical unit: the mean distance from the centre of the Sun to the centre of the Earth

   $1\ \text{au} = 1.496 \times 10^{11}\ \text{m}$

2. When a body (object floating in space) is orbiting another body, the primary force can be described using Newton's Law of Gravitation.

   $$F_G = \frac{G m_1 m_2}{r^2}$$

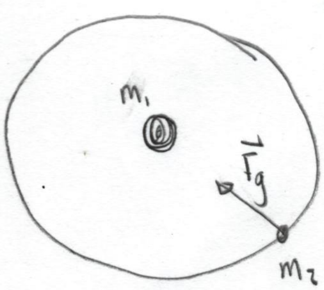
   constant $G = 6.67 \times 10^{-11}\ \frac{Nm^2}{kg^2}$

   $m_1 m_2$: mass of the 2 objects (kg)

   r: distance between centres of objects (m)

   F_G: force of gravity (N)

3. When one body has a circular orbit around another body, the orbital velocity can be calculated as follows:



$$\vec{F_g} = \vec{F_c}$$

$$\frac{G m_1 \cancel{m_2}}{r^2} = \cancel{m_2}\frac{v^2}{r}$$

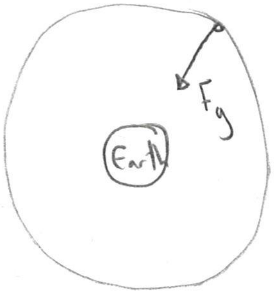
$$\frac{G m_1}{r} = v^2$$

$$v = \sqrt{\frac{G m_1}{r}}$$

$$r = \frac{G m_1}{v^2}$$

See over →

4.



$$\vec{F}_g = \vec{F}_c$$

$$\frac{F_{g\,orbit}}{F_{g\,surface}} = \frac{\dfrac{G\,m_{earth}\,m_2}{r_{orbit}^2}}{\dfrac{G\,m_{earth}\,m_2}{r_{Earth}^2}} = \frac{r_{Earth}^2}{r_{orbit}^2}$$

$$F_{g\,orbit} = F_{g\,surface} \times \left(\frac{r_{Earth}}{r_{orbit}}\right)^2$$

$$F_{g\,orbit} = \frac{m_2 v^2}{r_{orbit}}$$

$$F_{g\,surface} \times \left(\frac{r_{Earth}}{r_{orbit}}\right)^2 = \frac{m_2 v^2}{r_{orbit}}$$

$$m_2 g \times \left(\frac{r_{Earth}}{r_{orbit}}\right)^2 = \frac{m_2 v^2}{r_{orbit}}$$

$$g \times \frac{r_{earth}^2}{r_{orbit}} = v^2$$

$$v = r_{Earth}\sqrt{\frac{g}{r_{orbit}}}$$

4. When a satellite has a circular orbit around the Earth, the orbital velocity can be calculated as follows:

$$\vec{F}_g = \vec{F}_c$$

$$m_2 g_{orbit} = \frac{m_2 v^2}{r}$$

$$\frac{F_{g_{orbit}}}{F_{g_{earth}}} = \frac{\frac{\cancel{GM_{Earth}} \cancel{m_2}}{r_{orbit}^2}}{\frac{\cancel{GM_{Earth}} \cancel{m_2}}{r_{Earth}^2}} = \left(\frac{r_{Earth}}{r_{orbit}}\right)^2$$

$$F_{g_{orbit}} = F_{g_{earth}} \times \left(\frac{r_{Earth}}{r_{orbit}}\right)^2$$

$$F_{g_{Earth}}\left(\frac{r_{Earth}}{r_{orbit}}\right)^2 = \frac{v^2}{r_{orbit}}$$

$$mg \cdot \frac{r_{Earth}^2}{r_{orbit}} = v^2$$

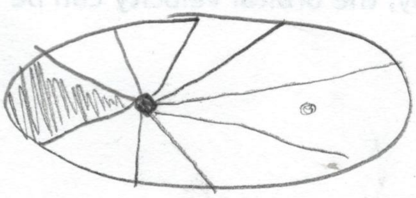
5. Kepplers's Three Laws of Planetary Motion

a. The Law of Ellipses:

Planets orbit the Sun in ellipses.

The Sun is at one of the focii.

b. The Law of Equal Areas:



Dividing an orbit into equal time periods, the shown areas will be equal.

c. The Law of Harmonics:

$$\frac{T^2}{r^3} = 1$$

T: Period in Earth years

r: radius of orbit in au

6. The Three Body Problem:

Once there are more than 2 bodies, we cannot solve the orbit analytically.

We have to model it with computers.

a. Animation of the problem

7. Urbain Le Verrier was a French mathematician who used the laws of planetary motion to predict the existence of Neptune.

See over →

Questions

1. A geostationary satellite has a period of 1.000 Earth days, such that it appears to "hover" permanently above the Earth. If the Earth has a radius of 6,371 km and a mass of 5.972×10^{24} kg, what is the orbital height of a geostationary satellite?
2. The International Space Station (ISS) has an orbital speed of 7.66 km/s. How high above the Earth's surface does it rotate?
3. The moon has an orbital velocity of 1.02 km/s. How far from the Earth's centre does the moon orbit?
4. If Mercury has a period of 0.241 Earth years, what is its average distance from the Sun?
5. If Pluto (no longer a planet) has a period of 248 Earth years, what is its average distance from the Sun?
6. If Jupiter has an average distance of 5.20 au from the Sun, what is its period?
7. If Uranus has an average distance of 19.18 au from the Sun, what is its period?

Answers

1. 35,900 km above the Earth
2. 418 km above the Earth
3. 383,000,000 km
4. 0.387 au
5. 39.4 au
6. 11.8 Earth years
7. 84.00 Earth years

1. $T = 1.000\ \text{day} \times \frac{24\ \text{hr}}{\text{day}} \times \frac{3600\ \text{s}}{\text{hr}} = 86{,}400\ \text{s}$

$d = 2\pi r_{orbit}$

$v = \frac{d}{T} = \frac{2\pi r_{orbit}}{T}$

$r_{orbit} = \frac{Gm}{v^2} = \frac{Gm}{\left(\frac{2\pi r_{orbit}}{T}\right)^2} = \frac{GmT^2}{4\pi^2 r_{orbit}^{\ 2}}$

$r_{orbit}^{\ 3} = \frac{GmT^2}{4\pi^2}$

$r_{orbit} = \sqrt[3]{\frac{GmT^2}{4\pi^2}} = \sqrt[3]{\frac{6.67\times10^{-11} \times 5.972\times10^{24} \times (86{,}400)^2}{4\pi^2}} = 42{,}231{,}000\ \text{m}$

$h = r_{orbit} - r_{earth} = 42{,}231{,}000\ \text{m} - 6{,}371{,}000\ \text{m} = 35{,}900{,}000\ \text{m} = \boxed{35{,}900\ \text{Km}}$

2. $r_{orbit} = \frac{Gm}{v^2} = \frac{6.67\times10^{-11} \times 5.972\times10^{24}}{(7{,}660)^2} = 6{,}789{,}000\ \text{m}$

$h = r_{orbit} - r_{Earth} = 6{,}789{,}000\ \text{m} - 6{,}371{,}000\ \text{m} = 418{,}000\ \text{m} = \boxed{418\ \text{Km}}$

3. $r_{orbit} = \frac{Gm}{v^2} = \frac{6.67\times10^{-11} \times 5.972\times10^{24}}{(1020)^2} = 383{,}000{,}000\ \text{m} = \boxed{383{,}000\ \text{Km}}$

4. $T^2 = r^3$

$$r = \sqrt[3]{T^2} = \sqrt[3]{0.241^2} = \boxed{0.387 \text{ au}}$$

5. $r = \sqrt[3]{T^2} = \sqrt[3]{248^2} = \boxed{39.5 \text{ au}}$

6. $T = \sqrt{r^3} = \sqrt{(5.20)^3} = \boxed{11.8 \text{ Earth years}}$

7. $T = \sqrt{r^3} = \sqrt{(19.18)^3} = \boxed{84.00 \text{ Earth years}}$