Mr. Renwick's Physics 12
Worksheet – Rotational Motion 3 – Banked Curves

Name: _______________
Block: _______________

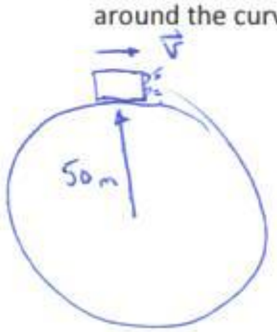
Learning Intentions

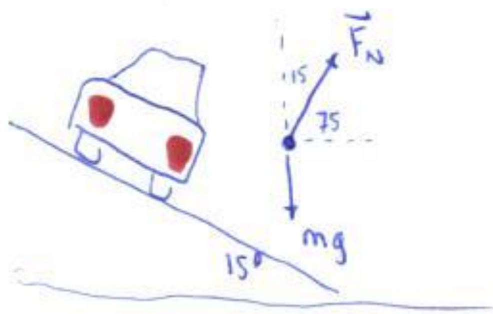
- Learn how to use centripetal acceleration to find the forces in objects moving in circles with banked curves.

Banked Curve Questions

In the design of highway roads, curves are often banked in order to help keep the car on the road.

1. A curve has a radius of 50 meters is banked at an angle of 15°. What driving speed would allow a car to go around the curve with no friction required between the car's tires and the road?





$$\Sigma F_y = ma_y = 0$$

$$F_N \sin 75 - mg = 0$$

$$F_N = \frac{mg}{\sin 75}$$

$$\Sigma F_x = ma_x$$

$$F_N \sin 15 = m \cdot a_c = m \cdot \frac{v^2}{r}$$

$$\frac{mg \sin 15}{\sin 75} = m \cdot \frac{v^2}{r}$$

$$v^2 = \frac{rg \sin 15}{\sin 75}$$

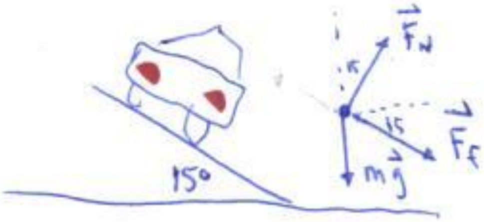
$$v = \sqrt{\frac{rg \sin 15}{\sin 75}} = \sqrt{\frac{rg \cos 75}{\sin 75}} \times \sqrt{\frac{\frac{1}{\cos 75}}{\frac{1}{\cos 75}}}$$

$$= \sqrt{\frac{rg}{\tan 75}} = \sqrt{\frac{50 \times 9.81}{\tan 75}}$$

$$= 11 \text{ m/s} \simeq 41 \text{ km/h}$$

$$\frac{\cancel{m}g\sin 15}{\sin 75} = \cancel{m}\cdot\frac{v^2}{r}$$

2. A curve has a radius of 50 meters and a bank angle of 15°. If the coefficient of friction between a car's tires and the road is 0.50, what is the maximum speed that the car can go around the curve?



$\Sigma F_y = 0$

③ $F_f = \mu F_N$

① $F_N \sin 75 - mg - F_f \cdot \sin 15 = 0$

$\Sigma F_x = ma_c = m\cdot\frac{v^2}{r}$

② $F_N \sin 15 + F_f \cos 15 = m\cdot\frac{v^2}{r}$

Sub ③ into ①

$F_N \sin 75 - mg - \mu F_N \sin 15 = 0$

$F_N(\sin 75 - \mu \sin 15) = mg$

$F_N = \frac{mg}{(\sin 75 - \mu \sin 15)}$ ④

See over →

~~Sub 2 into 1~~

Sub 3 into 2

5 $F_N \sin 15 + \mu F_N \cos 15 = m \cdot \frac{v^2}{r}$

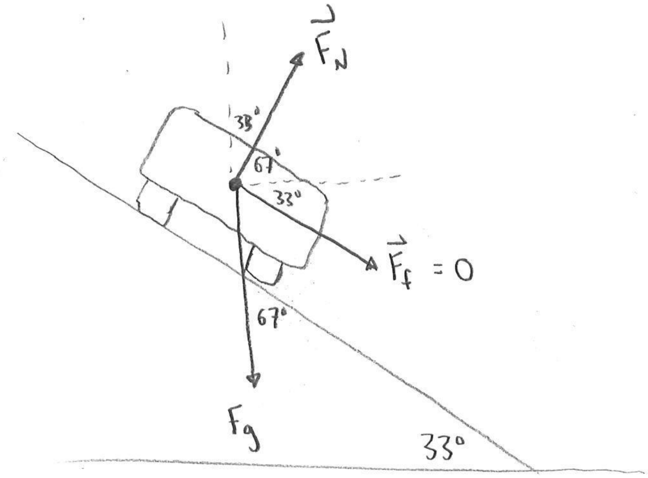
6 $F_N (\sin 15 + \mu \cos 15) = m \cdot \frac{v^2}{r}$

Sub 4 into 6

$$\frac{\cancel{m} g}{\sin 75 + - \mu \sin 15} (\sin 15 + \mu \cos 15) = \cancel{m} \cdot \frac{v^2}{r}$$

$$v = \sqrt{rg \cdot \frac{\sin 15 + \mu \cos 15}{\cos 15 - \mu \sin 15}} \approx 20.9 \text{ m/s} \approx 75 \text{ km/h}$$

3. $r = 1100\ ft \times \frac{0.3048m}{1ft.} = 335,28\ m$



$\Sigma F_y = 0$

$F_N \cos 33 - F_g = 0$

$F_N \cos 33 = F_g = mg$

$F_N = \frac{mg}{\cos 33}$

$F_c = F_{N_x} = F_N \sin 33 = \frac{mg \cdot \sin 33}{\cos 33} = mg \tan 33$

$F_c = ma_c$

$F_c = m \cdot \frac{v^2}{r}$

$\cancel{m} g \tan 33 = \cancel{m} \cdot \frac{v^2}{r}$

$v^2 = rg \tan 33$

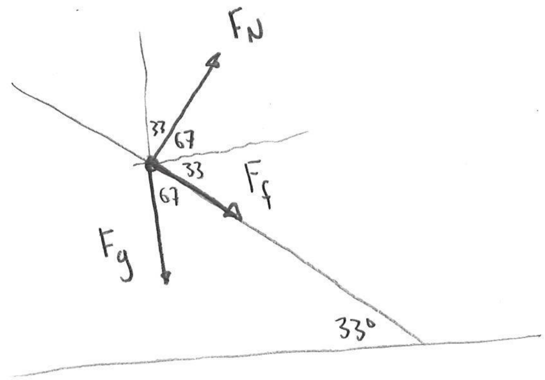
$v = \sqrt{rg \tan 33} = \sqrt{335.28m \times 9.81 m/s^2 \times \tan 33} = 46\ m/s = 170\ Km/h$

4. $F_f = \mu F_N$

$V = 320\ \text{Km/h} \times \frac{1\ \text{m/s}}{3.6\ \text{Km/h}}$

$F_g = mg$

$= 88.9\ \text{m/s}$



$\sum F_y = 0$

$F_N \cos 33 - F_f \sin 33 - mg = 0$

$F_N \cos 33 - \mu F_N \sin 33 = mg$

$F_N(\cos 33 - \mu \sin 33) = mg$

$F_N = \frac{mg}{\cos 33 - \mu \sin 33}$

$\sum F_x = ma_c = m \cdot \frac{v^2}{r}$

$F_N \sin 33 + F_f \cos 33 = m\frac{v^2}{r}$

$F_N \sin 33 + \mu F_N \cos 33 = m\frac{v^2}{r}$

$$\frac{\cancel{m}g}{\cos 33 - \mu \sin 33} \cdot \sin 33 + \mu \cdot \frac{\cancel{m}g}{\cos 33 - \mu \sin 33} \times \cos 33 = \cancel{m} \cdot \frac{v^2}{r}$$

$rg \sin 33 + \mu g r \cos 33 = v^2 \times (\cos 33 - \mu \sin 33) = v^2 \cos 33 - v^2 \mu \sin 33$

$\mu g r \cos 33 + v^2 \mu \sin 33 = v^2 \cos 33 - rg \sin 33$

$\mu(gr \cos 33 + v^2 \sin 33) = v^2 \cos 33 - rg \sin 33$

$\mu = \frac{v^2 \cos 33 - rg \sin 33}{v^2 \sin 33 + rg \cos 33} = 0.68$